Bearings:

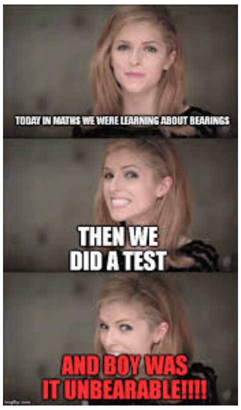


Table of Contents

This is a longsh worksheet to cater for the students that want extra practice. If you want a shortcut, but still be sure to cover one of each type then follow the pink highlighted questions.

Basics

Step By Step Method:

Step 1: Draw in any north lines ↑ at all the points if not already drawn in

Step 2: Connects the points if they aren't connected (make a path between them with a line)

Step 3: Locate which point the question says FROM and stick your pencil on the North line of this point

Step 4: Draw the angle (**clockwise!!!**) until you reach the point where the question says **OF**

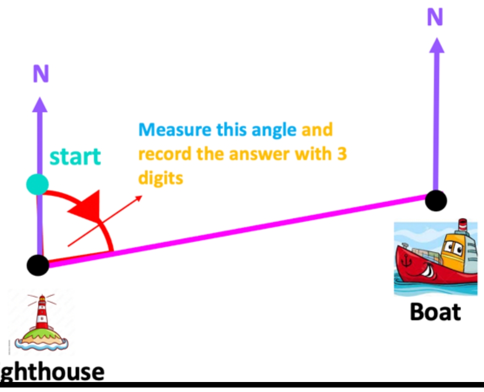
Step 5: There are 2 ways to proceed from here

- Easier Questions - If the question says measure/write down get your **protractor** and measure this angle
- Harder Questions – If the question says work out then use your **parallel line laws** (we treat the north Lines as our parallel lines - see next page)

Step 6: Give your answer to 3 significant digits
For example, if the angle is 20° you must write 020°

Example: Write down the bearing of the boat from the lighthouse below

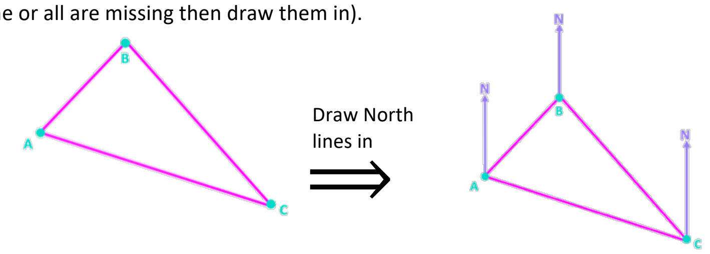
Answer
From lighthouse (start)
To boat (end)



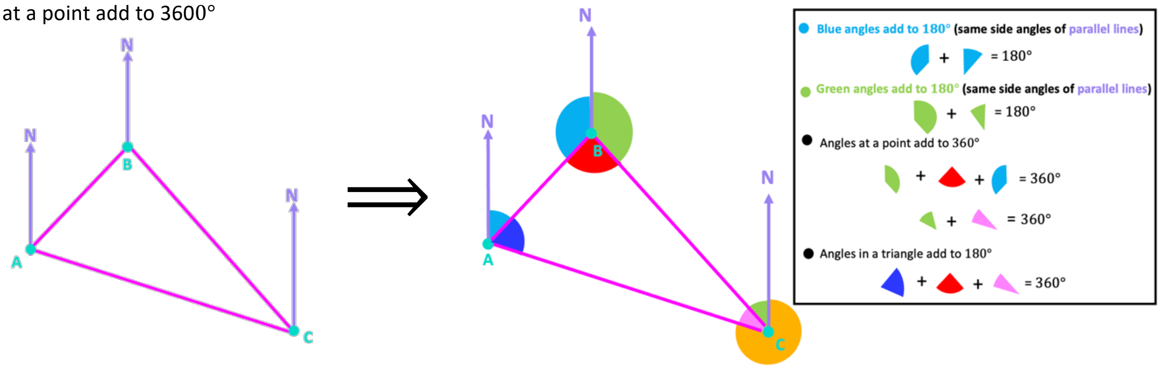
Step By Step Detailed Method

This topic is about finding angles which come from a North line and go in a clockwise direction! If the questions tell you to measure then you can use a protractor and you can skip step 2 below (this rarely comes up as it is too easy!) The question usually asks us to work out the angle, which means we cannot just use our protractor and measure the angles and hence cannot step 2 below.

- **Step 1:** Draw in North lines at all the points if any points are missing a North line (often all the points have North lines are already drawn for you in the diagram given, but if some or all are missing then draw them in).



- **Step 2:** Fill in all the missing angles that you can by using:
  - parallel line laws (the North lines are parallel lines). We use same side/co-interior angles which add to 180°.
  - Sum of angles in a triangle is 180°
  - Angles at a point add to 3600°



Note: Sometimes you will be told the triangle is isosceles (2 lengths are equal) which means the base angles are equal

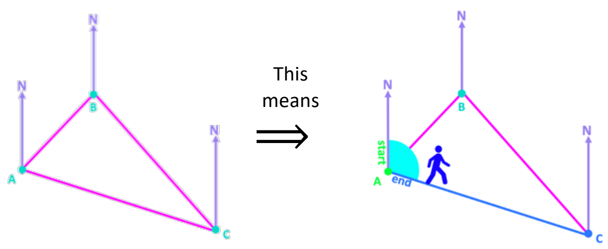
- **Step 3:** Locate the point where the question says **'from'** and the point where it says **'of'** (the question will always give a FROM point and an OF point). We always start at the North line of the point where it says **'from'** and go in a clockwise direction until we hit the line that would lead us (if we walked along that line) to the point where the question says **'of'**. This is the angle we want. Let's look at 2 examples:

Example 1: find the bearing of C from A

Firstly, we locate our 'from'; and 'of' points:

- Our 'from' point is A and our 'of' point is C.

This means we start at the North line of A (since from A) and go clockwise until the line that would leads us to C if we walked along the line (since of C).



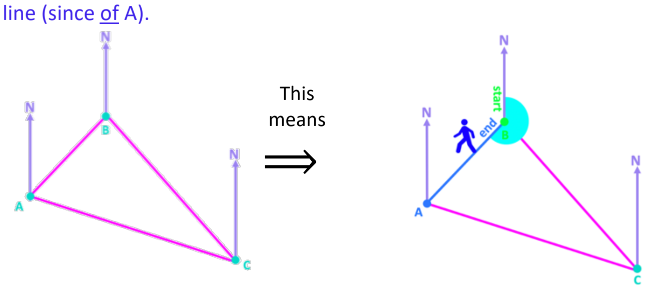
The turquoise angle is the bearing (angle) we want

Example 2: find the bearing of A from B

Firstly, we locate our 'from' and 'of' points:

- Our 'from' point is B and our 'of' point is A.

This means we start at the North line of B (since from B) and go clockwise until the line that would leads us to A if we walked along the line (since of A).



The turquoise angle is the bearing (angle) we want

- **Step 4:** Read off your answer. Always give your answers to 3 significant figures e.g. 40° = 040°

Harder Types of Questions:
You'll not always be given enough information (meaning that using parallel line laws is not enough to find all angles). You'll need to use your trig knowledge of **SOHCAHTOA/Sine Rule/Cosine Rule** to find the angles (obviously you'll need to have covered these topics first).

Drawing Your Own Diagram

You will often also have **draw** everything including **finding your own triangle** if the question is given in words without a diagram. We first draw the angle(s) given (remembering to start at a North line and go clockwise) with the correct distance(s) given and then form the triangle. See the silver section onwards for these types of questions.

Given 1 route:

- Start from **first point**, draw a **North line** and the **angle accurately**. Remember to go clockwise for bearings. Write in a **length** of the line if given.
- Draw a **north line** at the **second point**
- **Form** the **triangle** (looking for the **right angle**).
  Note: The question mentioning North, South, West, East is your hint for the right angle.
- Use **angles of $90°$ to fill in any missing angles**
- **Extract** the **triangle** and use **SOHCAHTOA**

Given 2 routes:

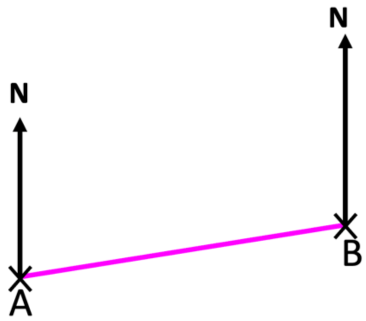
- Start from **first point**, draw a **North line** and the **angle accurately**. Remember to go clockwise for bearings. Write in a **length** of the line if given.
- Draw a **north line** at the **second point** and again draw the **angle accurately**. Write in a **length** of the line if given.
- Draw a **north line** at the **final point**
- **Form** the **triangle**
- Use the following angle facts to fil in missing angles
    - **parallel line laws co-interior/same side** (the north lines act as the parallel lines)
    - **angles add to $360°$**
    - **straight lines angles add to $180°$**
- **Extract** the **triangle** and use **SOHCAHTOA** (if you see a right angle) or **sine/cosine rule**

1 Bronze

1.1 Given Diagram

1.1.1 Measuring and Scale Diagrams

1) The diagram shows the positions of two telephone masts, A and B, on a map



i. Measure the bearing of B from A

Another mast C is on a bearing on 160° from B

On the map, C is 4 cm from B

ii. Mark the position of C with a cross (×) and label it C

2) The diagram shows an accurate scale drawing of part of the boundary of a field. The complete boundary

AB = 300 metres.

BC = 230 metres.

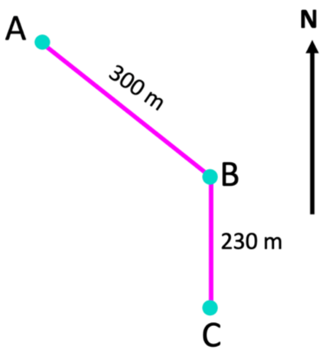
Point B is due north of point C.

The scale of the diagram is 1 cm to 50 metres. The bearing of D from C is 260°

AD = 480 metres.

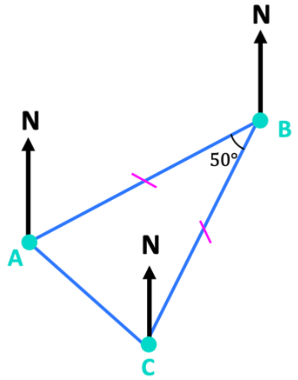
Complete the scale drawing of the boundary of the field.

Mark the position of D



1.1.2 Calculations

3) The diagram shows the positions of three points A, B, C, on a map. The bearing of B from A is 070°. Angle ABC is 50°. AB=CB.

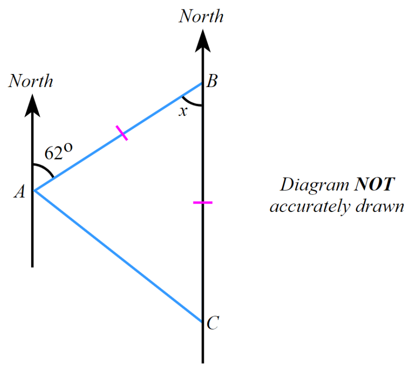


Work out the bearing of C from A.

1.1.3 Using Parallel Line Rules

4) Martin and Janet are in an orienteering race. Martin runs from checkpoint A to checkpoint B, on a bearing of 065°. Janet is going to run from checkpoint B to checkpoint A. Work out the bearing of A from B.

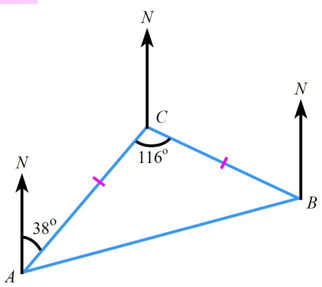
5) The bearing of B from A is 062°



C is due South of AB=CB

i. Find the size of angle x
ii. Give a reason for your answer
iii. Work out the bearing of C from A

6) The diagram shows three towns, A, B and C



Angle ACB=116°. CA=CB
Work out the bearing of

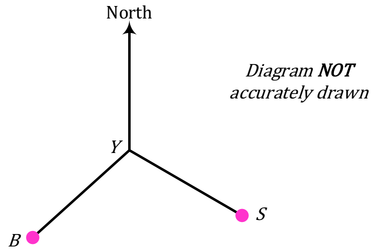
i. B from A
ii. B from C
iii. A from C

2 Silver

2.1 Given Diagram

2.1.1 Using Parallel Line Rules

7) The diagram shows the positions of a yacht Y, a ship S and a Beacon B. The bearing of B from Y is 228°



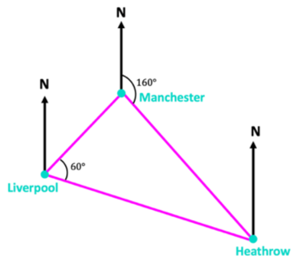
i. Find the bearing of Y from B

The bearing of S from Y is 118°

ii. Find the size of angle BYS

iii. Given also that BY=SY, find the bearing of S from B

8) The diagram shows the position of three airports, Heathrow, Manchester and Liverpool.

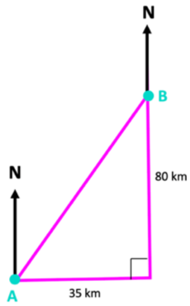


i. If Heathrow's bearing from Manchester is 160°, what is Manchester's bearing from Heathrow?

ii. If Liverpool's bearing from Manchester is 244°, what is Heathrow's bearing from Liverpool.

2.1.2 Using SOHCAHTOA

9) Town B is 35 km east and 80 km north of town A. Work out the bearing of A from B.



2.2 Drawing Your Own Diagram

2.2.1 Using SOHCAHTOA

10) A helicopter has flown from its base on a bearing of 153°. Its distance east of base is 19 km. How far has the helicopter flown?

11) A helicopter leaves its base and flies 23 km on a bearing of 285°. How far west is it from its base?

12) A plane flies 250 km on a bearing of 050°
    i. How far North is it from its original position
    ii. How far East is it from its original position

13) A ship sails on a bearing of 300° for 100 km. The captain can then see a lighthouse due south of him that he knows is due west of his starting point. Calculate how far west the lighthouse is from the ships starting point.

14) A ship at A is 3.8 km due North of a lighthouse. A ship at B is 2.7 km due east of a same lighthouse. What is the bearing of the ship at B from the ship at A?

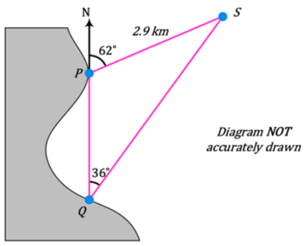
15) A fishing boat leaves port and sails on a straight course. After 2 hours its distance South of port is 24 km and its distance east of port is 7 km. On what bearing did it sail?

3 Gold

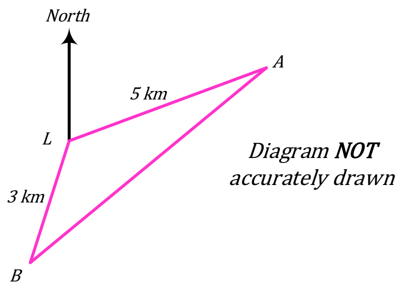
3.1 Given Diagram

3.1.1 Using Sine and Cosine Rule

16) P and Q are two points on a coast
P is due North of Q
A ship is at the point S
PS=2.9km
The bearing of the ship from P is 062°
The bearing of the ship from Q is 036°
Calculate the distance QS
Give your answer correct to 3 significant figures

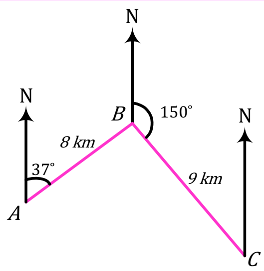


17) The diagram shows the position of two ships, A and B, and a lighthouse L



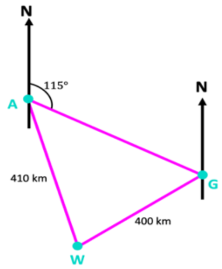
Ship A is 5 km from L on a bearing of 70° from L
Ship B is 3 km from L on a bearing of 210° from L
Calculate the distance between ship A and ship B
Give your answer correct to 3 significant figures

18) The diagram shows the position of three towns Acton (A), Barston (B) and Chorlten (C)



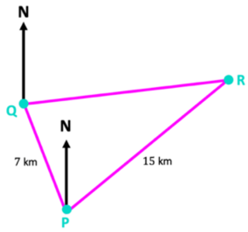
Barston is 8 km from Acton on a bearing of 037°. Chorlten is 9 km from Barston on a bearing of 150°. Find the bearing of Chorlten form Acton to the nearest whole number.

19) A plane flies from Auckland (A) to Gisborne (G) on a bearing of 115°. The plane then flies on to Wellington (W) on a bearing of 232°.



i. Calculate the size of angle AGW
The distance from Wellington to Gisborne is 400 kilometres
The distance from Auckland to Wellington is 410 kilometres
ii. Calculate the bearing of Wellington from Auckland

20) The diagram shows the position of three boats, P, Q and R. Boat Q is 7 km from boat P on a bearing of 327°. Boat R is 15 km from boat P on a bearing of 041°.



i. Find the distance between boats Q and R to 1 decimal place.
ii. Find the 3-figure bearing of boat R from boat Q.

3.2 Drawing Your Own Diagram

3.2.1 Using SOHCAHTOA

21) Jayne sails 1.5 km on a bearing of 050°. She then changes course and sails 2 km on a bearing on 140°. On what bearing must she sail to return to her starting position?

3.2.2 Using Sine and Cosine Rule

22) A boat sails from point X to point Y and then to point Z
Y is on a bearing of 280° from X
Z is on a bearing of 220° from Y
The distance from X to Y is 3.5 km
The distance from Y to Z is 6 km
Work out the bearing of Z from X

23) Chris ran 4 km on a bearing on 036° from P to Q. He ran in a straight line from Q to R, where R is 7 km due east of P. Chris then ran in a straight line from R to P. Calculate the total distance that Chris ran.

24) A helicopter flies on a bearing of 200° from A to B where AB=70 km. It then flies on a bearing of 150° from B to C, where C is due South of A. Work out the distance from C to A

3.2.3 Using SOHCAHTOA Twice

25) A yacht sails 15 km on a bearing of 053°, then 7 km on a bearing of 112°. How far North is the yacht from its starting position.

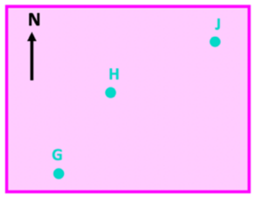
26) A plane flies 307 km on a bearing of 234°, then 23 km on a bearing of 286°. How far South is the plane from its starting position?

4 Diamond

4.1 Drawing Your Own Diagram

4.1.1 Using SOHCAHTOA

27) An aeroplane sets off from G on a bearing of 024° towards H, a point 250 km away. At H, it changes course and heads towards J on a bearing of 055° and a distance of 180 km away.
   i. How far is H to the north of G
   ii. How far is H to the east of G
   iii. How far is J to the north of H
   iv. How far is J to the east of H?
   v. What is the shortest distance between G and J?
   vi. What is the bearing of G from J?



4.1.2 Using Sine and Cosine Rule

28) Two radar stations A and B are 16 km apart and A is due North of B. A ship is known to be on a bearing of 150° from A and 10 km from B. Show that this information gives two positions for the ship, and calculate the distance between these 2 positions

29) A ship P sails from point A at noon in a direction of 35 degrees East of North, at 20 knots. One hour later, a second ship Q, sails from A in a direction 80 degrees East of North. At 3 PM, P is due North of Q.
   i. Calculate the speed of Q

   The ship P remains at that position, while Q sails on the same speed and in the same direction for one hour.
   ii. What is the bearing of Q from P at the end of the hour?

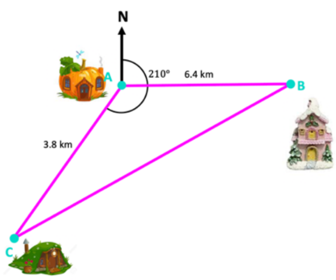
4.1.3 Using SOHCAHTOA and/or Sine and Cosine Rule

30) A ship and a helicopter depart from the same place. The ship sailed for 4 km on a bearing on 038°.
   i. How far east has it travelled?

   The ship is 4 km from the coast and sees a lighthouse. The angle of elevation from the ship to the lighthouse is 12°
   ii. How far above sea level is the lighthouse?

   Meanwhile, the helicopter flies 7 km on a bearing of 139°
   iii. How far away is the helicopter from the ship?

31) A, B and C are 3 villages
   B is 6.4 due East of A
   C is 3.8 km on a bearing of 210° from A
   Calculate the bearing of B from C
   Give your answer to the nearest degree



Bearings Solutions:

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1 Bronze

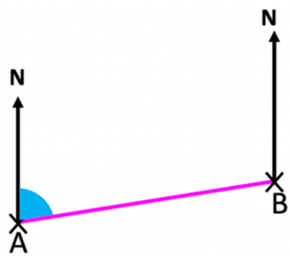
1.1 Given Diagram

1.1.1 Measuring/Scale Diagrams

1)

This is considered an easy bearings question since the question wants us to **measure** the angles. We can use our protractor and ruler. We don't have to use parallel line laws to work out the angles or trig knowledge such as SOHCAHTOA or sine/cosine rules to work out the angles (or side lengths).

i.



of B from A:

"from" is where we start (A) and "of" is where we end (B)

- "from": We always start at the North line of the correct point (in our case A) and we always go in a clockwise direction
- "to": we stop once we reach the line that leads us to this point (in our case B)

So, we start at the North Line of A and go in a clockwise direction until we reach the line that leads us to B.

This is the blue angle indicated in the diagram above

Measuring this with a protractor (since we are allowed in this question) gives 80°

Important: We always need to give our answer with 3 digits (to 3 significant figures)

080°

ii.

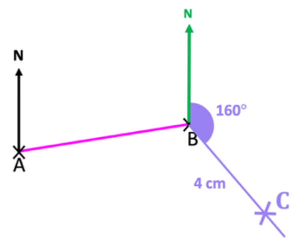
of C from B:

"from" is where we start (B) and "of" is where we end (C)

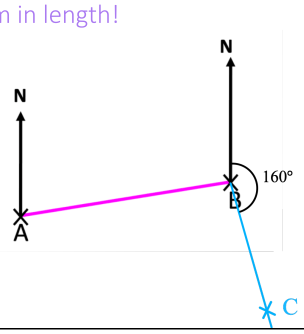
- "from": We always start at the North line of the correct point (in our case B) and we always go in a clockwise direction
- "to": we stop once we reach the line that leads us to this point (in our case C)

So, we start at the North Line of B and draw 160° clockwise which will give us the angle of the line for C.

We are told that the line is 4 cm long which we measure with a ruler starting from point C.



Note: The line above might not be 4 cm due to the shrinking of the diagram to fit this box, but yours should be 4 cm in length!



2)

This is also also considered an easy bearings question since we are given an **accurate scale drawing.** We can use our protractor and ruler. We don't have to use parallel line laws to work out the angles or trig knowledge such as SOHCAHTOA or sine/cosine rules to work out the angles (or side lengths).

The bearing of D from C is 260°:

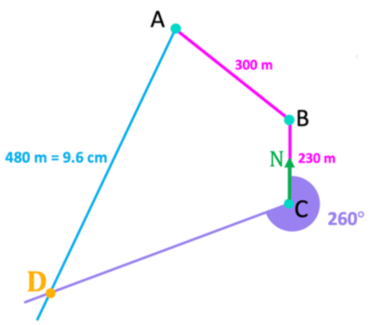
"from" is where we start (C) and "of" is where we end (D)

- "from": We always start at the North line of the correct point (in our case C) and we always go in a clockwise direction
- "to": we stop once we reach the line that leads us to this point (in our case D)

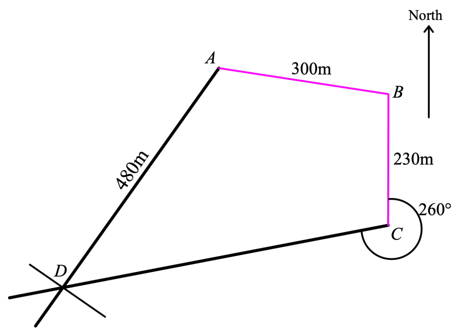
So, we **start** at the **North line** and draw an angle of 260° going **clockwise**

AD = 480 metres. The scale of the diagram is 1 cm to 50 metres, hence 480 m$= \frac{480}{50} = 9.6$ cm

Draw a line of length 9.6 and see where it intersects the purple line



Let's simplify this diagram now without all the colours

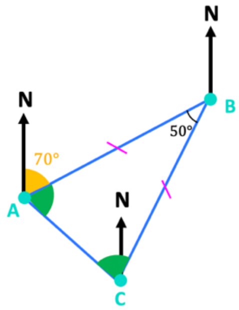


Note: The line above might not be 9.6 cm due to the shrinking of the diagram to fit this box, but yours should be!

3)

Here we have to **work out** the angle, so we cannot measure it like in the questions above which tell us to measure or give us an accurate scale drawing!

All the North lines are displayed at every point already, so we don't need to draw them in.



First let's work out any missing angles that we can. We have an isosceles triangle which can help us find the base angles of the triangle.
ABC is an isosceles triangle whose internal angles add up to $180°$ therefore,

$$\text{Angle BAC} = \frac{180-50}{2} = 65°$$

We are told the bearing of B from A is $070°$. This is the orange angle above. Why?
"from" is where we start and "of" is where we end.
So, we start at A (since says FROM A)
We go in a clockwise direction from the North line of where we start until where we end
Hence, we go clockwise from the North line of A until we hit the line that leads us to B (since says TO B).
This is the orange angle.

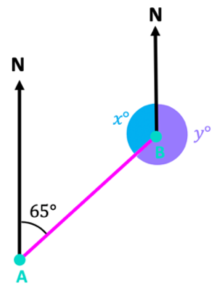
We want the bearing of C from A. This means start at the North line of A (since from A) and end at the line which leads us to c (since to C) making sure to go in a clockwise direction.

$$70 + 65 = 135°$$

1.1.2 Using Parallel Line Rules

4)

All the North lines are displayed at every point already, so we don't need to draw them in.



First let's work out the missing angles. To do this we use parallel line laws (the 2 North lines are the parallel lines and this is the key to working out angles with bearing questions along with SOHCAHTOA and sine/cosine rule which you will see later on in this sheet).

Same side interior angles add to $180°$ (C-angles)

$$x = 180 - 65 = 115°$$

Angles at a point add to 360°

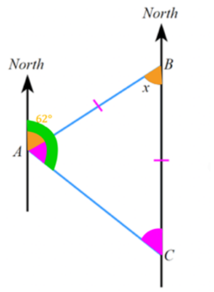
$$y = 360 - 115 = 245°$$

This question asks for the bearing of A from B. This is the purple angle above. Why?
"from" is where we start and "of" is where we end.
So, we start at B (since says FROM B)
We go in a clockwise direction from the North line of where we start until where we end.
Hence, we go clockwise from the North line of B until we hit the line that leads us to A (since says TO A).
This is the purple angle

$$245°$$

5)

All the North lines are displayed at every point already, so we don't need to draw them in.



i. $x = 062°$ (explanation is given in part ii.)

ii. The North lines are parallel lines.
Alternate angles (Z-angles) are equal

iii.
ABC is an isosceles triangle whose internal angles add up to $180°$ therefore,

BCA$= \frac{180-62}{2} = 59°$

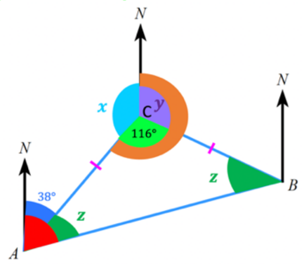
Bearing of C from A
"from" is where we start and "of" is where we end. We go in a clockwise direction from the North Line of A. So, we start at A and go until we hit the line that leads us to C.
This is the green angle

$59 + 62 = 121°$

6)

All the North lines are displayed at every point already, so we don't need to draw them in.

We are given that angle $ACB = 116°$



Let's first work out as many angles as we can before we work out the bearings

$x = 180 - 38 = 142°$ (since same-side angles add to $180°$)

$z = \frac{180-116}{2} = \frac{64}{2} = 32°$ (since base angles of an isosceles triangle are equal)

$y = 360 - 142 - 116 = 102°$ (angles at a point add to $360°$)

i. of B from A
"from" is where we start and "of" is where we end
Hence, we start at A and go until we hit the line that leads us to B
We go in a clockwise direction from the North Line of A
This is the red angle
$38 + 32 = 70°$

ii. of B from C
"from" is where we start and "of" is where we end
Hence, we start at C and go until we hit the line that leads us to B
We go in a clockwise direction from the North Line of C
This is the purple angle$102°$

iii. of A from C

"from" is where we start and "of" is where we end

So, we start at C and go until we hit the line that leads us to A

We go in a clockwise direction from the North Line of C

This is the orange angle

$$102 + 116 = 218°$$

2 Silver

2.1 Given Diagram

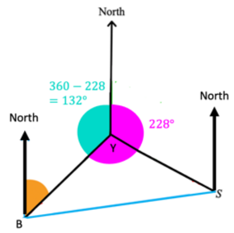
2.1.1 Using Parallel Line Rules

7)

i.

Not all the North Lines are drawn in here(we only need the North line at B to answer this part of the question, but let's get into a good habit of drawing them all in).

Let's work out as many angles as we can before we work out the bearings.



We are given that the bearing of B from Y is 228°

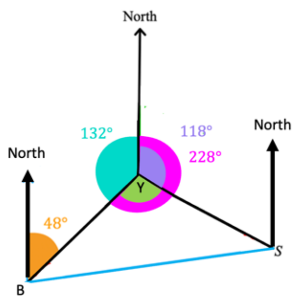
Angles at a point add to 360°

$$360 - 228 = 132°$$

048° (since the North lines are parallel lines and alternate interior angles are equal)

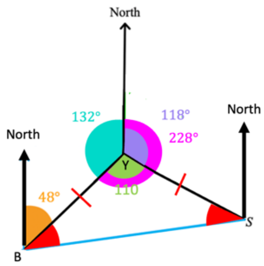
ii.

We are given that the bearing of S from Y is 118



Angle BYS = $228 - 118 = 110°$

iii.



Let's concentrate on the triangle (we found the angle BYS from part ii. above)

BY=SY therefore, BYS is an isosceles triangle whose internal angles add up to $180°$ therefore,

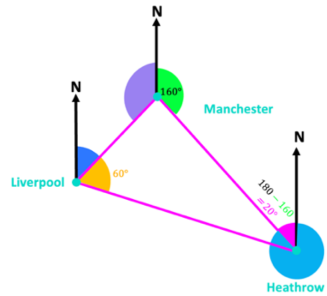
$$\frac{180-110}{2}=35°$$

of S from B "from" is where we start and "of" is where we end. We go in a clockwise direction from the North Line of where we start (B). So, we start at B and go until we hit the line that leads us to S.

$$48+35=083°$$

8)

i.

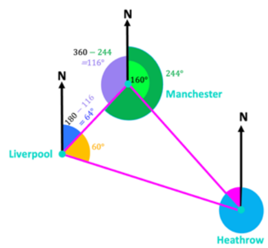


- $180-160=20$ (same side angles with parallel lines)
- $360–20=340$ (angles at a point add to 360)

We are asked for of Manchester from Heathrow
"from" is where we start and "of" is where we end
We go in a clockwise direction from the North Line of Heathrow
So, we start at Heathrow and go until we hit the line that leads us to Manchester

This is the light blue angle= $340°$

ii.



- $360-244=116$ (angles at a point add to 360)
- $180-244=64$ (same side angles add to 180)
- 60 (given)

of Heathrow from Liverpool
"from" is where we start and "of" is where we end
We go in a clockwise direction from the North Line of Liverpool
So, we start at Liverpool and go until we hit the line that leads us to Heathrow
This is the dark blue plus the orange angle

$$64+60=124°$$

2.1.2 Using SOHCAHTOA

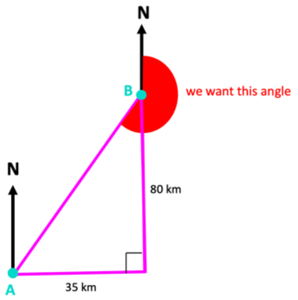
9)

This question relies on knowledge of another trigonometry topic – SOHCAHTOA. If you are not familiar or comfortable with this topic, see my worksheet 'SOHCAHTOA' first.

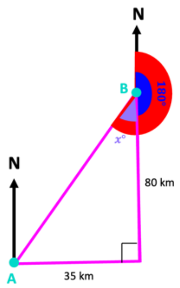
All the North lines are displayed at every point already, so we don't need to draw them in.

We want the bearing of A from B "from" is where we start and "of" is where we end
So, we start at B and go until we hit the line that leads us to A
We go in a clockwise direction from the North Line of B

This means we want the red angle



This red angle below which made up of the straight line blue angle 180° plus the purple angle which we can find using SOHCAHTOA since we have a right angled triangle



We need angle x before we even worry about tackling the bearings part

$$\tan x = \frac{opp}{adj} = \frac{35}{80}$$

$$x = \tan^{-1}\left(\frac{35}{80}\right) = 23.6°$$

Now let's find the bearing which is the red angle

$$\text{Bearing} = 180 + 23.6 = 204°$$

2.2 Drawing Your Own Diagram

2.2.1 Using SOHCAHTOA

10)

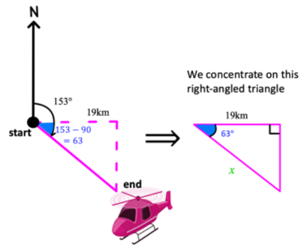
We now have to draw our own diagram as we are not given it

Way 1: Form the right angle triangle below the line

We draw the bearing of 153° from the north line of the starting point.

We know the distance East is 19 km and can label this

We can form a right-angled triangle (dashed lines) since the distance is an <u>east</u> distance



$$153 - 90 = 63°$$

Consider the right-angled triangle

$$\cos 63 = \frac{19}{x}$$

$$x = \frac{19}{\cos 63}$$

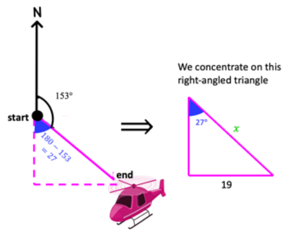
$$x = 41.851 \ldots = 41.9\text{km}$$

Way 2: Form the right angle triangle above the line

We draw the bearing of 153° from the north line of the starting point

We know the distance East is 19 km and can label this

We can form a right-angled triangle (dashed lines) since the distance is an <u>east</u> distance



$$180 - 153 = 27°$$

Consider the right-angled triangle

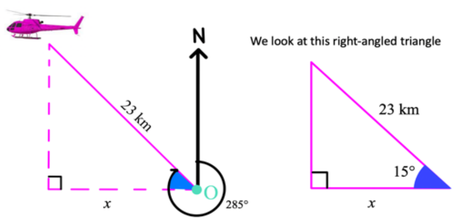
$$\sin 27 = \frac{19}{x}$$

$$x = \frac{19}{\sin 27}$$

$$x = 41.851 \ldots = 41.9\text{km}$$

11)

Way 1: Form the right angle triangle below the line



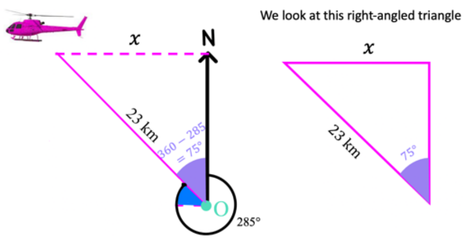
$$285 - 270 = 15°$$

$$\cos 15 = \frac{x}{23}$$

$$x = 23 \cos 15$$

$$x = 22.2 \text{ km}$$

Way 2: Way 1: Form the right angle triangle above the line



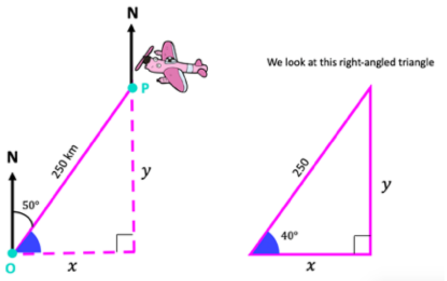
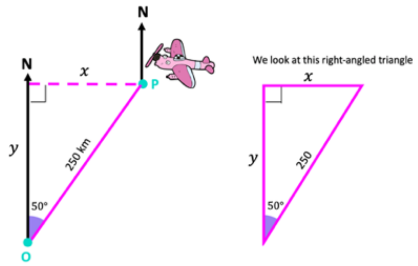
$$\sin 75 = \frac{x}{23}$$

$$x = 23 \sin 15$$

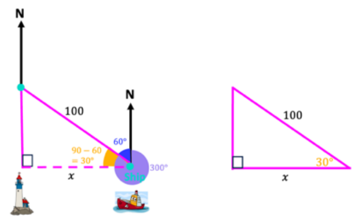
$$x = 22.2 \text{ km}$$

12)

i.

Way 1: Form the right angle triangle below the line	Way 2: Form the right angle triangle above the line
 $90 - 50 = 40°$ i. $\sin 40 = \frac{y}{250}$ $y = 250 \sin 40$ $y = 161$ km ii. $\cos 40 = \frac{x}{250}$ $x = 250 \cos 40$ $x = 192$ km	 i. $\sin 50 = \frac{x}{250}$ $x = 250 \sin 50$ $x = 192$ km ii. $\cos 50 = \frac{y}{250}$ $y = 250 \cos 450$ $y = 161$ km

13)

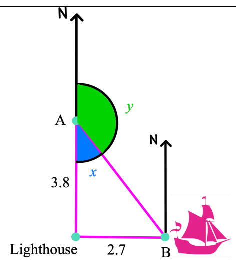


$$\cos 30 = \frac{x}{100}$$

$$x = 100 \times \cos 30$$

$$x = 86.6 \text{ km}$$

14)



Consider the pink triangle

$$\tan x = \frac{2.7}{3.8}$$

$$x = 35.3947 \ldots$$

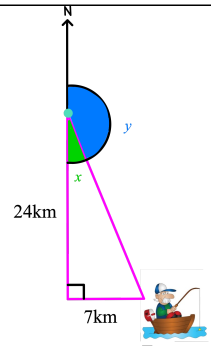
Hence,

$$y = 180 - 35.3947 \ldots$$

$$y = 144.605 \ldots$$

$$y = 145°$$

15)



$$\tan x = \frac{7}{24}$$

$$x = 16.2602\ldots$$

Hence,

$$y = 180 - 16.2602\ldots$$

$$y = 163.739\ldots$$

$$y = 164°$$

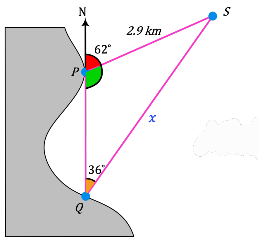
3 Gold

3.1 Given Diagram

3.1.1 Using Sine and Cosine Rule

16)

This question relies on knowledge of another trigonometry topic – Sine and Cosline Rule. If you are not familiar or comfortable with this topic, see my worksheet 'Sine and Cosine Rule' first.



Angles on a straight line add to $180°$

$$180 - 62 = 118°$$

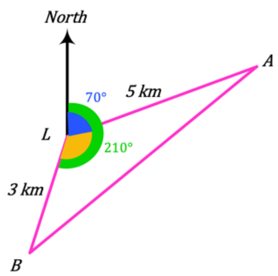
Now use the sin rule

$$\frac{2.9}{\sin 36} = \frac{x}{\sin 118}$$

$$x = \frac{2.9 \sin 118}{\sin 36}$$

$$x = 4.36 \text{ km (3sf)}$$

17)

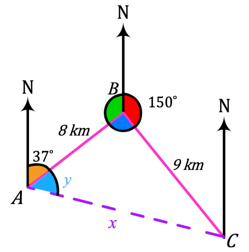


$$210 - 70 = 140°$$

Now we can calculate the distance between A and B using the cos rule.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = 3^2 + 5^2 - 2(3)(5) \cos 140$$
$$a^2 = 56.981 \ldots$$
$$a = 7.5485 \ldots$$
$$a = 7.55\ km$$

18)



Same side/co-interior angles add to 180°

Therefore,

$$180 - 37 = 143°$$

Angles at a point add up to 360°

$$360 - 150 - 143 = 67°$$

Now we can calculate x using the cos rule.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 8^2 + 9^2 - 2(8)(9)\cos 67$$
$$x^2 = 88.734\ldots$$
$$x = 9.4199\ldots$$

Now use the sin rule to find y

$$\frac{\sin 67}{9.4199} = \frac{\sin y}{9}$$

$$\sin y = \frac{9\sin 67}{9.4199}$$

$$y = \sin^{-1}\left(\frac{9\sin 67}{9.4199}\right)$$

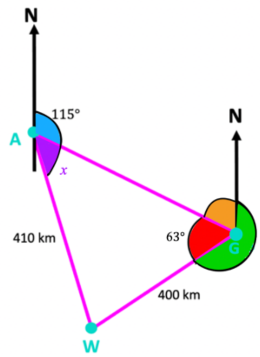
$$y = 61.5786\ldots$$

$$y = 61.6$$

Therefore, the bearing of Chorlten from Acton

$$37 + 61.6 = 099°$$

19)



i.

Same side/co-interior angles add to 180°. Therefore,

$$180 - 115 = 65°$$

Angles at a point add up to 360°. Hence,

$$360 - 232 - 65 = 63°$$

ii.

Use the sin rule to find x

$$\frac{\sin x}{400} = \frac{\sin 63}{410}$$

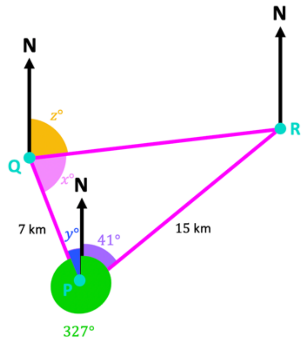
$$\sin x = \frac{400\sin 63}{410}$$

$$x = \sin^{-1}\left(\frac{400 \sin 63}{410}\right)$$
$$x = 60.3744\ldots$$
$$x = 60.4$$

Hence,

$$115 + 60.4 = 175°$$

20)



i.

Angles at a point add to 360°

$$y = 360 - 327 = 33°$$

Therefore, angle QPR,

$$33 + 41 = 74°$$

Now we use the cos rule to calculate the distance between Q and R

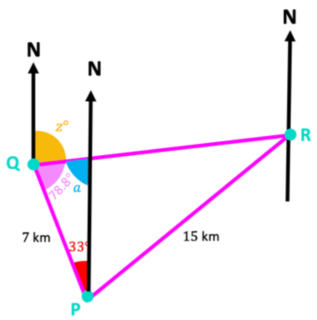
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$QR^2 = 7^2 + 15^2 - 2(7)(15) \cos 74$$
$$QR^2 = 216.116\ldots$$
$$QR = 14.700\ldots$$
$$QR = 14.7\text{km}$$

ii.

$$\frac{15}{\sin x} = \frac{14.7}{\sin 74}$$

$$\sin x = \frac{15 \sin 74}{14.7}$$

$$x = \sin^{-1}\left(\frac{15 \sin 74}{14.7}\right)$$
$$x = 78.777 = 78.8°$$



Considering the small triangle (internal angles of a triangle add to 180°)

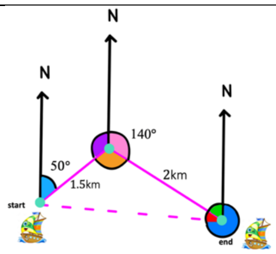
$$a = 180 - 78.8 - 33 = 68.2°$$

Alternate angles (z angles) are equal, therefore, the bearing of boat R from boat Q

$$z = 068°$$

3.2 Drawing Your Own Diagram

21)



The question wants the blue angle

Let's find all angles first

Same side/co-interior angles add to 180°

$$180 - 50 = 130°$$
$$180 - 140 = 40°$$

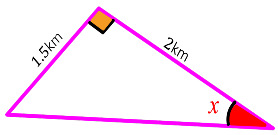
Angles at a point add to 360°

$$360 - 130 - 140 = 90°$$

We have a right-angled triangle hence we use SOHCAHTOA

We want the blue angle, but need the red angle to be able to find this

Consider the right-angled triangle



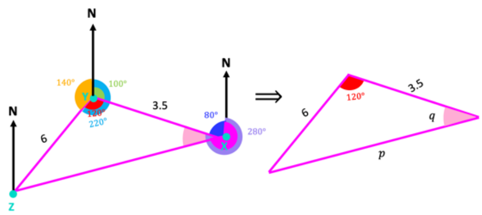
$$\tan x = \frac{1.5}{2}$$
$$x = 36.8698\ldots$$
$$x = 36.9$$

Angles at a point add to 360°. Hence,

$$360 - 36.9 - 40 = 283°$$

3.2.1 Using Sine and Cosine Rule

22)



The dark pink angle is the bearing we want. Do find this we need the light pink angle

$$p^2 = 6^2 + 3.5^2 - 2(6)(3.5)\cos 120$$

$$p = 8.321658$$

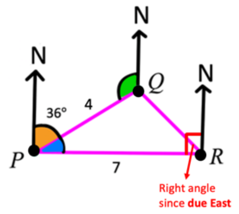
$$\frac{\sin q}{6} = \frac{\sin 120}{8.321658}$$

$$\sin q = 0.6244$$

$$q = 38.639°$$

$$280 - 38.639 = 241.4°$$

23)

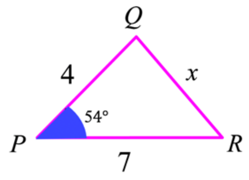


Same side interior angles add to $180°$ (C-angles)

$$180 - 36 = 144°$$

And NPR is 90 since the line PR is horizontal (R is due East)

$$90 - 36 = 54°$$



Now we use the cos rule to find x

$$x^2 = 4^2 + 7^2 - 2(4)(7)\cos 54$$
$$x^2 = 32.084\ldots$$
$$x = 5.6642\ldots$$
$$x = 5.66$$

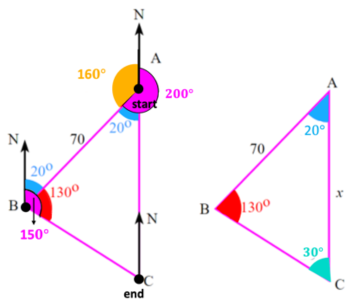
Therefore, the total distance

$$4 + 7 + 5.66 = 16.66 = 16.7 \text{ km}$$

24)

The initial bearings at A and B are shown in pink on the diagram below

C is **due south** hence it is directly below A



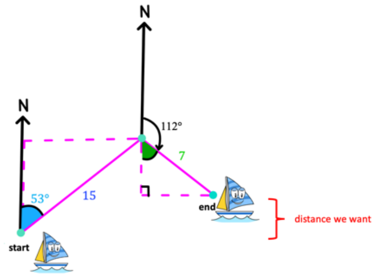
$$\frac{70}{\sin 30} = \frac{x}{\sin 130}$$

$$x \times \sin 30 = 70 \times \sin 130$$

$$x = \frac{70 \times \sin 130}{\sin 30}$$
$$x = 107.246\ldots$$
$$x = 107\ km$$

3.2.2 Using SOHCAHTOA Twice

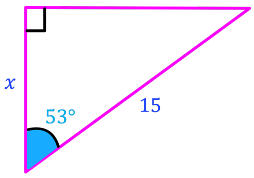
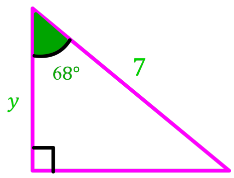
25)



This question is not like usual. We are not looking to find the distance between the start and finish point and hence we don't need to connect the start and finish point.

We are asked how far **North.** This is our hint to use SOHCAHTOA.

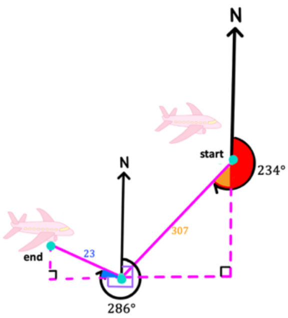
Consider both triangles formed

Triangle on the left	Triangle on the right
 $\cos 53 = \frac{x}{15}$ $x = 15\cos 53$ $x = 9.027227$	$180 - 112 = 68$  $\cos 68 = \frac{y}{7}$ $y = 7\cos 68$ $y = 2.622246$

The difference between the lengths will give us the North length we want

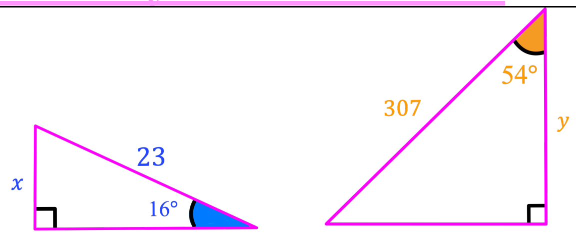
$9.027 - 2.622 = 6.40$ km

26)



$234 - 180 = 54°$
$286 - 270 = 16°$

Now consider the right-angled triangles



$$\sin 16 = \frac{x}{23}$$
$$x = 23 \sin 16$$

$$\cos 54 = \frac{y}{307}$$
$$y = 307 \cos 54$$

Now consider the South components as vectors and add them up to find how far South the plane is from its origin. If South is positive then North is negative.

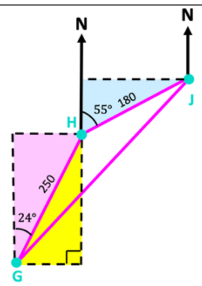
$$307 \cos 54 - 23 \sin 16 = 174 \text{ km}$$

4 Diamond

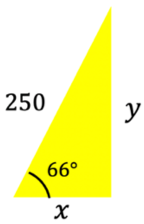
4.1 Drawing Your Own Diagram

4.1.1 Using SOHCAHTOA

27)



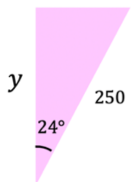
i.



$\sin 66 = \frac{y}{250}$

$y = 228.4\ km$

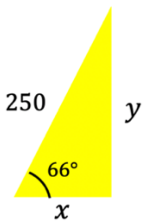
Or we can look at the pink triangle



$\cos 24 = \frac{y}{250}$

$y = 228.4\ km$

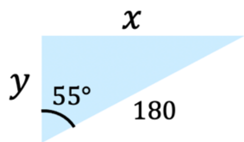
ii.



$\cos 66 = \frac{x}{250}$

$y = 101.7\ km$

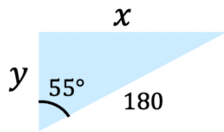
iii.



$\cos 55 = \frac{y}{180}$

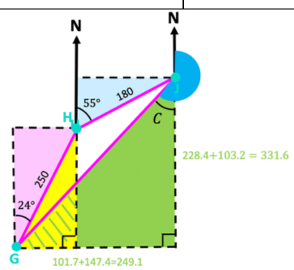
$y = 103.2\ km$

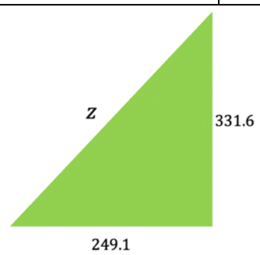
iv.



$\sin 55 = \frac{x}{180}$

$y = 147.4\ km$





v. $z^2 = 331.6^2 + 249.1^2$

$z = 415$ km

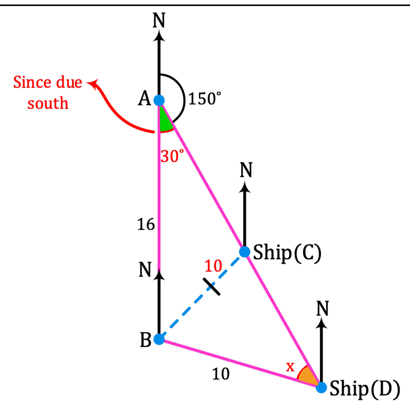
vi. $\tan c = \frac{249.1}{331.6}$

$c = 36.9°$

$180 + 36.9 = 219.6 = 217°$

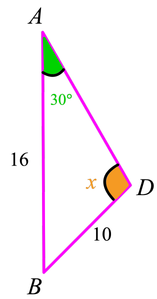
4.1.2 Using Sine and Cosine Rule

28)



Note: Ship can be either North or South of point B since we don't know the distance from A to the ship. This also shows the ambiguity in the sine rule.
Hence there are 2 possible positions for ship (C or D)
Lengths must be the same from B though since ship is a fixed distance from B hence 1 isosceles triangle.

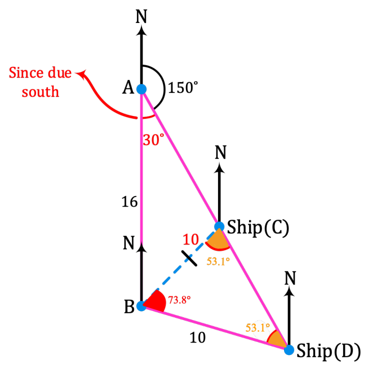
Looking at triangle ABD



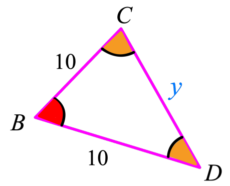
$$\frac{\sin x}{16} = \frac{\sin 30}{10}$$

$$\sin x = \frac{16 \sin 30}{10} = 53.1° \text{ or } 180 - 53.1 = 126.87° \text{ due to ambiguous case of the sine rule}$$

So, we now have



Now let's look at triangle BCD



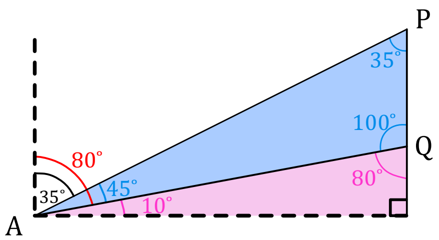
$$\frac{y}{\sin 73.8} = \frac{10}{\sin 53.1}$$

$$y = \frac{10 \sin 73.8}{\sin 53.1} = 12.0$$

Therefore, ships are 12km apart.

29)

i.
We can draw this a seen below since P is due North of Q. Let's also work out all the angles that we can.

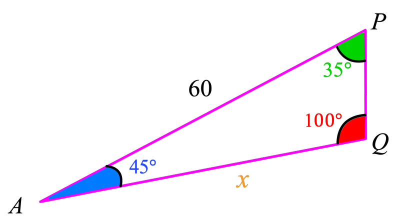


P goes for 3 hours
Q goes for 2 hours

$$\text{Speed} = 20$$
$$D = S \times T$$

So,

$$D = 20 \times 3 = 60 \text{ for P}$$



$$\frac{\sin 100}{60} = \frac{\sin 35}{x}$$

So

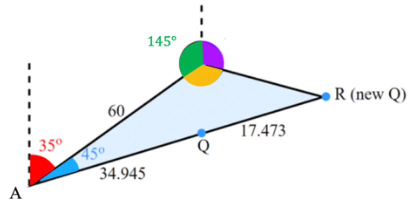
$$x = 34.945$$

Now we want the speed for Q

$$S = \frac{D}{T} = \frac{34.945}{2} = 17.5 \text{ knots}$$

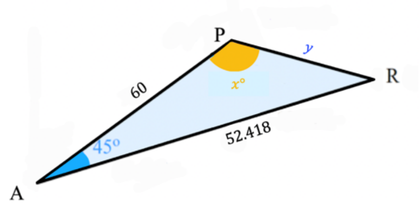
ii.

Consider



Note: Q travels for 3 hours now. We know that QR=17.473 (same distance since same direction and only 1 hour so we divide by 2)

We need Angle APR



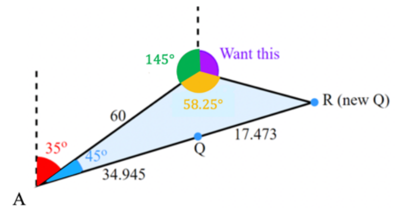
Using the cos rule

$$y^2 = 60^2 + 52.418^2 - 2(60)(52.418)\cos 45$$
$$y = 43.587$$

Now use the sin rule

$$\frac{52.418}{\sin x} = \frac{43.587}{\sin 45}$$
$$\sin x = \frac{52.418 \sin 45}{43.587}$$
$$x = 58.25°$$

Now consider



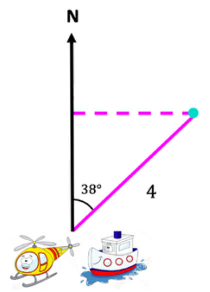
Angles at a point add to $360°$

$$360 - 145 - 58.25 = 157°$$

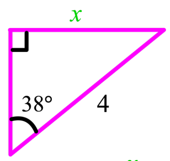
4.1.3 Using SOHCAHTOA and Sine and Cosine Rule Together

30)

i.

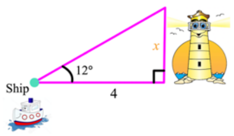


Looking at the triangle



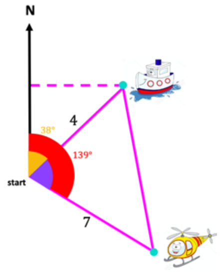
$$\sin 38 = \frac{x}{4}$$
$$x = 4 \sin 38 = 2.46 \text{ km}$$

ii.

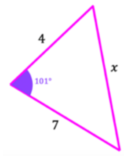


$$\tan 12 = \frac{x}{4}$$
$$x = 4\tan 12 = 0.850\text{ km} = 850\text{m}$$

iii.



Looking at the following triangle:
$139 - 30 = 101°$

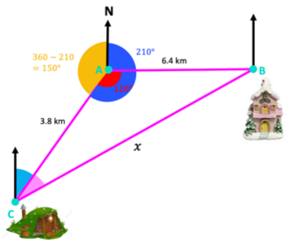


Not a right-angled triangle so can't use SOHCAHTOA
Use cos rule instead

$$x^2 = 4^2 + 7^2 - 2(4)(7)\cos 101$$
$$x^2 = 75.685 \ldots$$
$$x = 8.70\ km$$

31)

Way 1: using cosine rule (recommended)



We want the light blue plus pink angle

$$x^2 = 3.8^2 + 6.4^4 - 2(3.8)(6.4)\cos 120$$
$$x^2 = 79.72$$
$$x = 8.92861$$

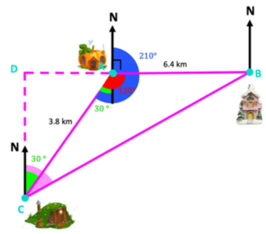
$$\frac{\sin C}{6.4} = \left(\frac{\sin 120}{8.92861}\right)$$

$$\sin C = 6.4\left(\frac{\sin 120}{8.92861}\right)$$
$$C = \sin^{-1}(0.62076)$$
$$C = 38.37197$$

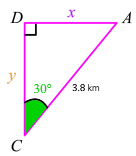
Angles at a point add to 360°
$360 - 210 = 150°$

Same side/co-interior angles add to 180°
$180 - 150 = 30°$
$30 + 38.37197 = 68.37 = 068°$

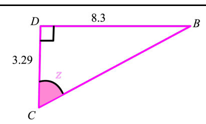
Way 2: using SOHCAHTOA (harder)



Consider the following right-angled triangles



$\sin 30 = \frac{x}{3.8}$	$\cos 30 = \frac{y}{3.8}$
$x = 3.8\sin 30$	$y = 3.8\cos 30$
$x = 1.9$	$y = 3.29$



$$\tan z = \frac{6.4 + 1.9}{3.29}$$

$$z = \tan^{-1}\left(\frac{8.3}{3.29}\right) = 68.4$$

Therefore, the bearing

$$= 068°$$